Multiplicative relationships and their visual representations
Relations multiplicatives et leurs représentations visuelles

Word problems can be defined as verbal descriptions of problem situations wherein one or more questions are raised, the answer to which can be obtained by the application of mathematical operations to numerical data available in the problem statement. […] Typically a student who is confronted with the problem in the context of a mathematics lesson or a mathematics test is required to give a numerical answer to a specific question by making explicit and exclusive use of the quantities given in the text and mathematical relationships between those quantities inferred from the text. (Verschaffel, Greer, & De Corte, 2000, p. ix)

Davydov (1982) describes the concept of quantitative relationship as a low "by which the relation between two elements determines a unique third element as a function" (p. 229)

In his seminal work, Davydov (1982) put forward the idea of quantitative relationships as mathematical concepts that we need to teach and learn in elementary school, even prior to numbers. He argues that the concept of number appears from the multiplicative comparison of two magnitudes (or quantities), one playing the role of unit of measurement and the other being measured.

We make a clear distinction between the relationship governing a situation and a solution strategy for a particular problem based on the situation. We suggest that a solution strategy should be derived from the relationship and not from the particularity of the problem. (example: the train story presents a grouping situation, but if the number of wagons is to be found, the operation to use would be division and not multiplication)

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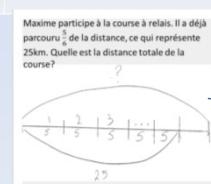Marie-Sophie Gélinas, CSVT
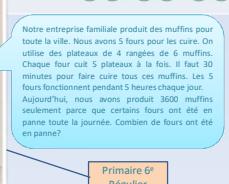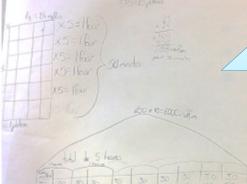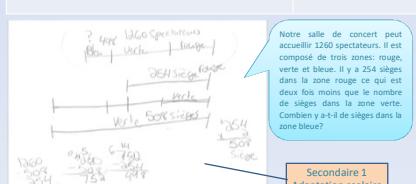Claudine Gervais, CSDGS

Mathematically Impossible Situation

Il y a 24 passagers dans le train.
Le train est composé de 12 voitures.
Dans chaque voiture, il y a 6 passagers.

The **multiplicative relationship** between three values is corrupted.

There are 24 passengers in the train. The train has 12 wagons. There are 6 passengers in each wagon.

Terminology used in school and literature	Problem	Representation	Relationship according to Davydov theory	Representation
Repeated addition / Addition répétée	Gustave receives 3 objects each day. How many objects will he have in 4 days? / Gustave reçoit 3 objets par jour pendant 4 jours. Combien d'objets a-t-il reçus?		Total quantity measured = 12 objects / Unit = 3 objects per day / Number of repetitions (or coefficient) = 4 / Unit of reference = 1 object	Quantité totale = 12 objets / Unité = 3 objets / Nombre de répétitions (coefficient) = 4 / Unité de base = 1 objet
Sharing or partition / Partage	Gustave share 12 objects between 4 people. How many objects each person will have? / Gustave partage 12 objets entre 4 personnes. Combien d'objets chaque personne doit-elle recevoir?		Total quantity measured = 12 objects / Unit = ? objects per person / Number of people (coefficient)= 4 / Unit of reference = 1 object	3 objects — Repeated 4 times — Total of 12 objects
Measurement or quotition / Contenance	Gustave put 12 objects in bags, 3 objects per bag. How many bags will it make? / Gustave met 12 objets dans des sacs, 3 objets par sac. Combien de sacs va-t-il utiliser?		Total quantity measured = 12 objects / Unit = 3 objects per bag / Number of bags (coefficient)= ? / Unit of reference = 1 object	
Comparison / Comparaison	Gustave has 3 objects. Maude has 4 times more than Gustave. How many does she have? / Gustave a 3 objets. Maude a 4 fois plus d'objets que Gustave. Combien d'objets Maude a-t-elle?	4 times more	Total quantity measured = ? objects / Unit of measurement = 3 objects / Coefficient= 4 times more / Unit of reference = 1 object	3 objects — Repeated 4 times — Total of 12 objects
Rectangular disposition / Disposition rectangulaire	In the classroom, there are 4 rows of 3 desks each. How many desks are there in the classroom? / Dans la classe, il y a 4 rangées de 3 pupitres chaque. Combien de pupitres y a-t-il dans la classe?		Total quantity = 12 desks / Dimension 1 = 3 rows / Dimension 2 = 4 desks per row / Unit of reference = 1 desk	Total quantity = 12 Desks / Unit = 3 desks per row / Number of rows (coefficient)= 4 / Unit of reference = 1 desk
Cartesian product or area / Produit cartésien ou l'aire	Gustave has 3 pants and 4 shirts. How may different outfits can he make? / Gustave a 3 pantalons et 4 chemises. Combien de costumes différents (pantalon-chemise) peut-il faire?	Tree / Table	Total quantity = 12 outfits / Dimension 1 = 3 pants / Dimension 2 = 4 shirts / Unit of reference = 1 outfit	4 shirts — Total of 12 outfits — 3 pants