Problem solving game: A teaching approach

Purpose

This game of problem solving has been designed to engage students in a self-directed approach to learning mathematics. However, the role of the teacher in this learning is crucial because this software does not teach problem solving per se. Rather, it creates conditions for students to test their knowledge, identify gaps, and evaluate their progress. This means that the student takes on most of the responsibilities traditionally assigned to the teacher.

Structure

There are four categories of problems:

- Problems with additive structures (additive relations). These problems can be solved mainly by addition and subtraction;
- Problems with multiplicative structures (multiplicative relationships). These problems can be solved mainly by multiplication and division;
- Problems with various structures (relationships). Each problem of this group simultaneously describes at least one additive relationship and one multiplicative relationship. These problems can be solved by several operations—a combination of addition, subtraction, multiplication, and division.
- Problems with fractional expressions. In the statements of these problems, we find expressions of fractions (for e.g. "a third of," "three quarters").

In each category, there are problems of three levels of complexity. The level of difficulty of each problem does not necessarily match its level of complexity. At each level of complexity, there are relatively easy or difficult problems. However, the complexity levels correspond to the learning progress. Here is the reference chart.

Age	Addition			Multiplication			Variation			Fractions		
	1	2	3	1	2	3	1	2	3	1	2	3
6-8 years	v	v		v			v					
8-10 years	v	v	v	v	v		v	v		v		
10-12 years		v	v		v	v		v	v	v	v	
12+			v			v		v	v	v	v	v

Using this table, the teacher can refer students to any category of problems. However, it is suggested that the decision on the challenge to be overcome is made by the student.

Less complex problems are grouped by context and by the relationship they describe. Problems of the same group have the same name and are distinguished by a letter. Example: the problems Lucie a), Lucie b) and Lucie c) describe the same situation but present different questions.

a) Lucie has 5 dresses and 14 socks. She buys 3 dresses. How many dresses does she have now?
b) Lucie has some dresses and 14 socks. She buys 5 dresses. Now she has 13 dresses. How many dresses did she have at first?
c) Lucie has 4 dresses and 14 socks. She buys some dresses. Now she has 10 dresses. How many dresses did she buy?

This grouping of problems that present the same relationship (structure) makes it possible to value the learning of the relationship (structure) rather than of particular strategies of resolution, and often allows students to generalize the relationship itself rather than a particular operation.

To support this generalizable learning about relationships (structures), we suggest the Equilibrated Development method as a teaching approach. This method is described in the following papers:

Représenter pour mieux analyser (coming soon)

Polotskaia, E., & Savard, A. (2018). Using the Relational Paradigm: effects on pupils' reasoning in solving additive word problems.*Research in Mathematics Education*, *20*(1). https://doi.org/10.1080/14794802.2018.1442740

Freiman, V., Polotskaia, E., & Savard, A. (2017). Using a computer-based learning task to promote work on mathematical relationships in the context of word problems in early grades. *ZDM Mathematics Education*, *0*(0), 1–15. https://doi.org/10.1007/s11858-017-0883-3

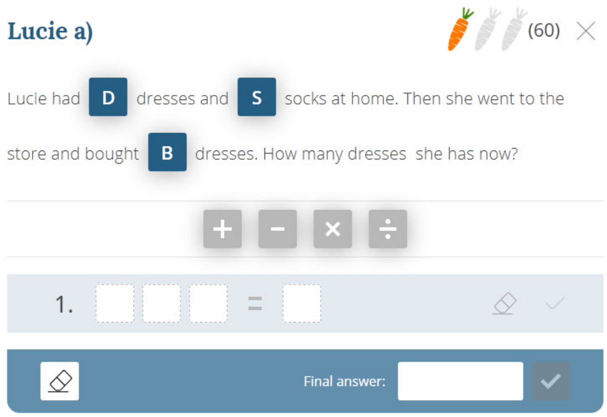
It is important to note that the problems included in the game are suitable for any approach to teaching problems. The choice of teaching approach is at the teacher's discretion.

To reinforce the effect of generalization, it is proposed to solve all the problems in a group together. An understanding of the situation developed for each problem may a) help to solve another problem, b) be deepened by each succeeding resolution.

The resolution game

As in any game, there are rules in the resolution game. The rules are there to enhance certain aspects of mathematics learning.

Each problem has a statement and a question. Digital data is hidden behind cards identified by letters. For example the problem Lucie a) is presented to the player in the following way:



The number 5 is behind the letter "R", the number 14 is behind the letter "B", and the number 13 is behind the letter "A". At any time, the player can make the number visible by double-clicking on the letter. He can solve the problem in a general way, by composing formulas (expressions with letters); or by calculation, by composing operations with numbers. In the latter case, the computer calculates the numerical response for each operation. The player loses a number of points in the game for each number he made visible. This rule in the game reinforces the value of generalization and reasoning about relationships between quantities.

The interface of the game is designed so that the player is obliged to solve the problem operation by operation. We cannot create equations. A numerical answer cannot be submitted if the number is not obtained by an operation and is not present on the screen. This particular feature in the game allows for the analysis of relationships, understanding of the mathematical structure of the situation to be solved, as well as the eventual transformation of this structure into a chain of arithmetic operations.

There is a limit to the number of operations in a solution. In the case of complex problems, this rule favors the search for a shortest, most economical solution.

Here are two typical activities that a teacher can use to create effective learning while respecting and developing students' autonomy, interest, and motivation.

Activity 1 (each student has access to a computer)

The educational objective: We would like students to try to test their knowledge in problem solving and identify their difficulties and knowledge gaps. Once a difficulty has been identified, the class-group, together with the teacher seeks solutions and develops knowledge to meet the student's needs. So students learn when they feel a need for learning. They actively appropriate the developed knowledge.

Teaching goal: There is a wide variety of problems and structures in the game. Each time, the teacher chooses a category of problems, she would work with her students on quantitative relationship that are additive, multiplicative, mixed, or with fractions. However, we cannot target a particular structure because several structures are present in the same category. For example, additive comparison problems cannot be specifically targeted because they are in the same category as add-on or take-back problems. Nevertheless, all these problems have an additive relationship and can be represented by similar schemes. We must therefore follow students' choice with regard to the particular structure, but we can concentrate the effort on learning a particular relationship.
The didactical goal is to deepen or develop the understanding of the relationship (for e.g., additive), and to develop metacognition to support the problem solving of this relationship.

Introduction: Students are invited to participate in a problem-solving competition. Students can play in teams, individually, or just accumulate points for fun. To participate, you must register (individually or create an account for each team). Registered players will be able to compare their accumulated points and decide who wins. Problems can be solved without any registration, but in this case the points will not be accumulated.

Students must be shown how to navigate the game page and how to choose problems. We then explain that to solve the problem, we must compose the mathematical expressions (operations) using each time two data and an operation sign. Each data can be a letter, a number, or a formula (expression) obtained by one of the preceding operations. Once the final expression or the numerical answer is obtained, players deposits it on the field of answers and confirms their choice. The explanatory videos are available on the pages of the game.

Chase to difficulties

Students are encouraged to try to solve the problems they want for about 20 minutes. It is wise to choose one category at a time to facilitate discussion afterwards.

Students are reminded of the method of working with the problems (if this method has been taught). There is no visual support on the computer to create various representations or resolutions. Students should use a sheet of paper and a pencil to create the representation they need.

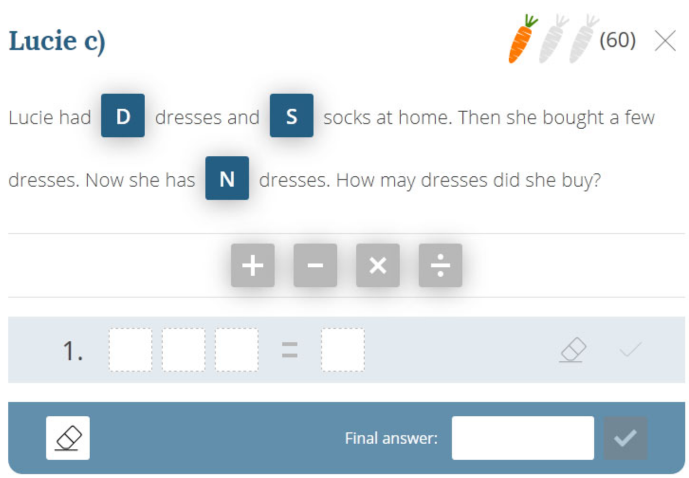
The students' task is to try a few problems and identify those that seem difficult, those that cannot be solved. After 20 minutes, students will report these problems (the name of the problem) and collectively the group will decide which issues to discuss together.

Discussion and learning

Once in a large group, we decide what problem to discuss. Let's assume Lucie (c) is selected because many students chose it. The teacher invites one of the students who identified a difficulty with the problem to come to the board and explains this student's role. The student at the board should listen carefully to the group's proposals and try to follow their instructions on the board. If unable to do so, the group should rephrase their proposal, find another explanation until the student who requested help is satisfied and is able to complete the task. Finding a solution to the problem or building a representation is not the responsibility of the student at the board, but that of the group. The group works to help the one who identified the problem.

Here is an example of a discussion (7-8 year olds).

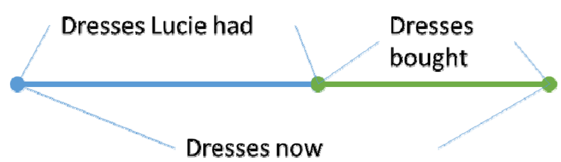
On the computer's screen:



Teacher: You have to explain to Marta how to represent the problem, how to build a schema to better understand the situation. She will listen to you and try to follow your suggestions.
Student 1: It's easy.
Teacher: It's not easy or difficult. We ask for exact explanations. Can you explain us?
Student 1: You just have to remove the N and you'll know the answer.
Teacher: Why?
Student 1: You just have to remove the N because he says, "How many dresses did she buy? "
Teacher [to Marta]: Did you understand? [Marta nods for "yes"] Me - no. I did not understand why I have to remove N?
Student 1: Because if you take N, you have an answer, how many dresses she bought.
Teacher: Why?
Student 1: Because there is the number. [The student wants to see the number behind the letter N.]
Student 2: Em ... It's like D dresses and S socks. That too we do not know.
Student 3: But socks, it does not tell us what's going on.

Teacher: Attention, we have some important information here. Marta, listen carefully. Student 3, you are telling us that we do not need to know how many socks. Is that what you are saying?
Student 3: Yes.
Teacher: Why?
Student 3: It says, "How many dresses did she buy? "
Teacher [to Marta]: What do you think? [Marta nods for "yes."] Do you think we do not need socks?
Marta: Because the question only asks for the dresses, we just take the dresses.
Teacher [to Student 3]: Bravos, thank you very much! You gave us a good explanation. [To Marta] Thank you?
Marta: Thank you.
Teacher: Here, the socks do not interest us, because the question tells us about dresses. So, it's not important how many socks there are. We do not need to open this box.

We continue the discussion to build a representation and then to find the relevant operation. Here is a possible representation for the Lucie-c problem).



The same representation is suitable for the problems Lucie a) and Lucie b).

The teacher can insist on a representation without numbers to reinforce students making generalizations. The teacher can "give in" to the students' desire and make visible numbers but discuss the relevance of each number for the solution.

When the problem is discussed and understood, it is solved on the computer to confirm it.

For more information, read the article:

Polotskaia, E., & Freiman, V. (2016). Technopédagogie et apprentissage actif. *Bulletin AMQ, LVI*(3), 55–69.

Activity 2 (if computers are not available to students)

The pedagogical objective: To lead students to develop a metacognitive view on the resolution of written problems.

Teaching goal: Have students compare written problems that present the same relationship and generalize this relationship.

Preparation: Print a collection of problems with the same name. For example, we print the problems Lucie a), Lucie b) and Lucie c) using letters. The younger the students, the simpler and shorter the text to read, and the fewer problems you will include in the group.

Introduction (hunt for difficulties): Students are told how the game works on the computer. It is important to emphasize that problems must be solved arithmetically, operation by operation. Students are encouraged to solve them in a general way, using letters. The three problems are distributed to each student or student team. Students are asked to read the problems and decide which of the three problems seems the most difficult.

Discussion and learning: Collectively, we choose a single problem. A discussion of this problem is conducted, similar to the discussion above. We make sure that this problem is well analyzed, represented, and understood by the students. When It is solved, it is checked on the teacher's computer.

Integration and Investment: Students (or teams) are encouraged to use their lived experience to resolve the remaining problems on the list. For each problem that remains, a team is asked to share its solution. The solution is checked using the computer. If the solution does not work, the problem is discussed collectively and rechecked on the computer.

The similarity and difference between the problems is discussed. Students are asked if solving the chosen problem helped them solve other problems. If so, their initial choice was relevant, otherwise, you have to choose better next time.